S.V.UNIVERSITY, MODEL PAPER.

THREE YEAR B.A/B.Sc DEGREE EXAMINATIONS.

CHOICE BASED CREDIT SYSTEM

III SEMESTER

PART II : MATHEMATICS

Paper III :ABSTRACT ALGEBRA

(New Syllabus w.e.f 2015-16)

Time: 3 hours **Max Marks :75**

SECTION - A

Answer any FIVE of the following questions. Each question carries 5 marks (5X5 = 25).

1. Show that the fourth roots of unity is an abelian group w.r.t multiplication.
2. Prove that identity element in a group is unique.
3. If Z is the additive group of integers, then prove that the set of all multiples of integers by a fixed number "m" is subgroup of Z.
4. Prove that intersection of two sub groups $H_1$and H_2 of group G,is a subgroup of G.
5. Show that H ={ 1,-1} is a normal subgroup of the group of non-zero real numbers under multiplication.
6. If G is a group of non-zero real numbers under multiplication the prove that $f(x) = x^2 : G \to G$ is a homomorphism. Determine Ker f.
7. Examine whether the following permutation is even or odd.

   $$\begin{pmatrix} 1 & 2 & 34 & 5 & 67 & 8 & 9 \\ 6 & 1 & 43 & 2 & 57 & 9 & 8 \end{pmatrix}$$

8. Define cyclic group and give an example.

(P.T.O)

SECTION - B

Answer ALL of the five questions. Each question carries 10marks (5X10 = 50).

9 a.Prove that the set-Z of all integers form an abelian group w.r.t the operations defined by

$a * b = a + b + 2$,$\forall$ a,b $\in$ Z.

OR

b. Show that the set G= { 1,2,3,4,5,6} is a finite abelian group of order 6 w.r.t X_7 .

10a. Prove that the necessary and sufficient condition for a complex H of a finite group G to be a subgroup is $\forall a, b \in H \Rightarrow ab \in H$.

OR

b. State and prove Lagrange's theorem.

11a. A subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

OR

b. Prove that a sub group H of a group G is normal subgroup of G iff each right coset of H in G is left coset of H in G.

12a. State and prove Fundamental theorem on homomorphism of groups.

OR

b. If $\emptyset : Z_{10} \rightarrow Z_{10}$ be a homomorphism defined by $\emptyset(1) = 8$,then find Ker $\emptyset$.

13a. State and prove Cayley's theorem.

OR

b. The order of a cyclic group is equal to the order of its generator.

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